

My talk this evening is about Tidal and Vertical Datums.

At the outset I need to say that many in this room of hydrographic surveyors may know more about one or both of these topics than I. I'm thinking especially about those who got their start in the Royal Navy.

On the other hand, there may be material here that will be new to some ... and there is a little math. Relax. This whole presentation can be downloaded at the URL shown on this slide. You can read at your leisure later what you miss tonight.

Motivation

- The real world is three-dimensional, therefore, coordinates are necessarily 3-D
- In the oil patch we carefully qualify horizontal coordinates with geodetic datums, map projections, grid parameters, et cetera, but not so much in the vertical
- Gravity defines the vertical (up / down) dimension in the real (non-ellipsoidal) world
- Careful qualification is as required in the vertical as in the horizontal for many reasons, one of which is the integrity of 3D visualization

2

The world is 3D.

In the oil industry we've been careful about horizontal coordinate references for several decades now, but only recently has that care extended to the vertical dimension.

Gravity defines the vertical in the real world.

We need to qualify the vertical for several reasons, among them Earth-Centered Earth-Fixed visualization, my passion.

Ellipsoidal Vertical



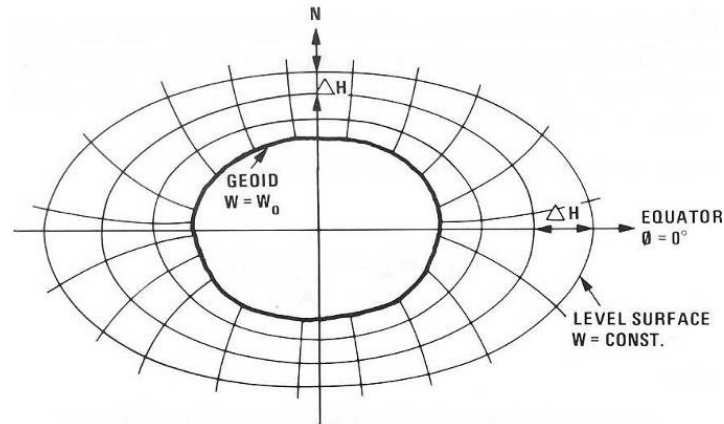
Horizontal graticule of parallels and meridians, latitudes and longitudes, intersecting orthogonally on the ellipsoid.

The ellipsoidal vertical is measured along the normal, the straight line perpendicular to the ellipsoid surface.

3

The ellipsoidal normal is one kind of vertical, a straight line.
The ellipsoid, however, is not a physical surface.

Gravitational Vertical



Level equipotential surfaces are neither parallel nor equally spaced.
Vertical lines (equipotential perpendiculars) are curved.

Elevation is measured with respect to the geoid, the surface whose potential (W) approximates that of Mean Sea Level ($W=W_0$).

4

This graphic from Gregory Hoar (see references) is central to our understanding of gravity-based vertical. It shows a number of equipotential surfaces around the Earth, like layers of an onion. A satellite in space will follow along in an equipotential surface, where the equipotential, or geopotential in the case of gravity (magnetism is another potential field), is every constant. That is W (or U or V depending upon author) is constant. Notice that the equipotential surfaces are neither parallel or equally spaced. ΔH in the Equatorial plane is different than ΔH above the pole. The vertical is everywhere perpendicular to the equipotential. Therefore, the gravity vertical is curved (unlike the ellipsoidal normal). The equipotential surface whose geopotential (W) is the same as the geopotential of Mean Sea Level (W_0) is called the geoid.

Now, how many times have you seen a diagram like this and wondered what geopotential was. Is it a force? No. An acceleration? No. What is it? What are its units?

Gravitational Geopotential (W)

If $W = g \cdot D$, where g is the acceleration of gravity ($\pm 9.78 \text{ m/s}^2$ on the geoid at the Equator) and D is the distance to the center of the Earth ($\pm 6,378,137 \text{ m}$ on the geoid at the Equator), then

$$\begin{aligned} W_0 &\approx 62\text{M m}^2/\text{s}^2 = 62\text{M Joules/kg} \\ &= 6.2\text{M kgal}\cdot\text{m} = 6.2\text{M GPU} \end{aligned}$$

Geopotential (W_0) is everywhere constant on the geoid, but the acceleration of gravity (g) is not!

Geopotential is important later in the presentation.

5

Hoar gives this formula for geopotential (W). It's the acceleration of gravity times the distance to the center of the Earth. Now, if we take the values of g and D at the Equator for the WGS84 ellipsoid, we can compute the geopotential of the geoid as 62 million meters squared per seconds squared. That's the value of the geopotential anywhere on the geoid. But meters squared per seconds squared? What's that? We can think of it as a velocity squared, or as an acceleration-meter. Strange units.

Another definition of the geopotential is the work done in a force field moving a particle of unit mass from infinity to the point it occupies. Work is force over a distance, or a Newton-meter, or a Joule in SI. For a unit mass we get Joule/kg, whose units are, in fact, meters squared per seconds squared.

At this point we introduce another unit that is central to several types of vertical datums that we will discuss, the Geopotential Unit (GPU), which is 10 meters squared per seconds squared. So, the geopotential of the geoid is 6.2 million GPU.

Neither the force of gravity nor the acceleration of gravity (what you measure with a gravity meter) are constant on the geoid.

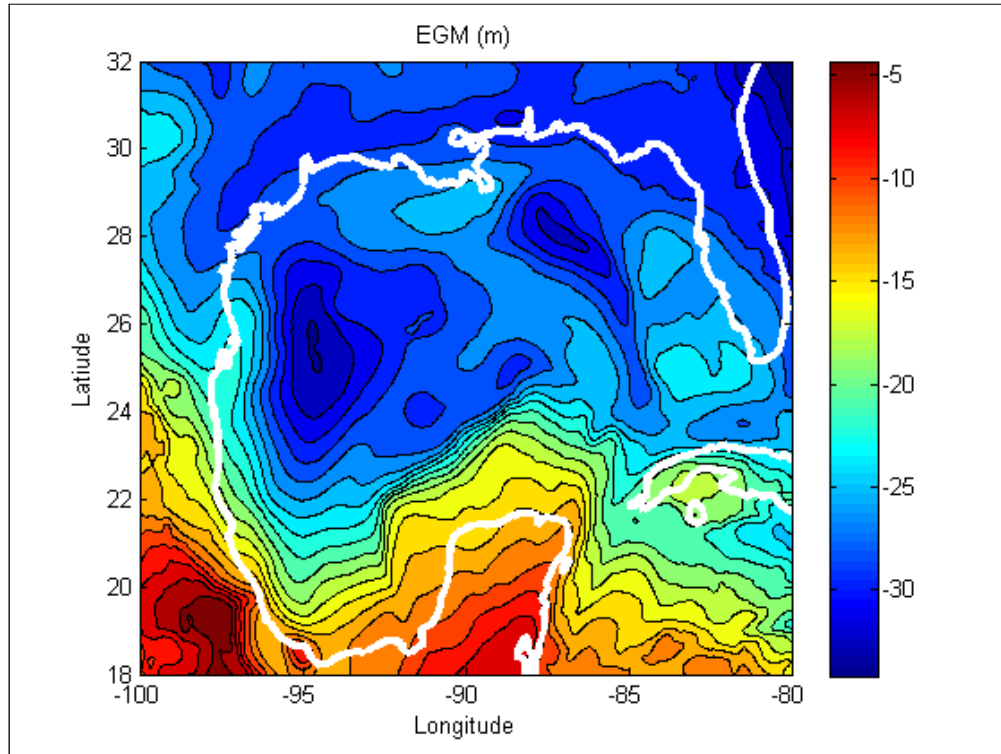
Mathematically, the vector g is the gradient of the scalar W .

Introducing Tidal Datums

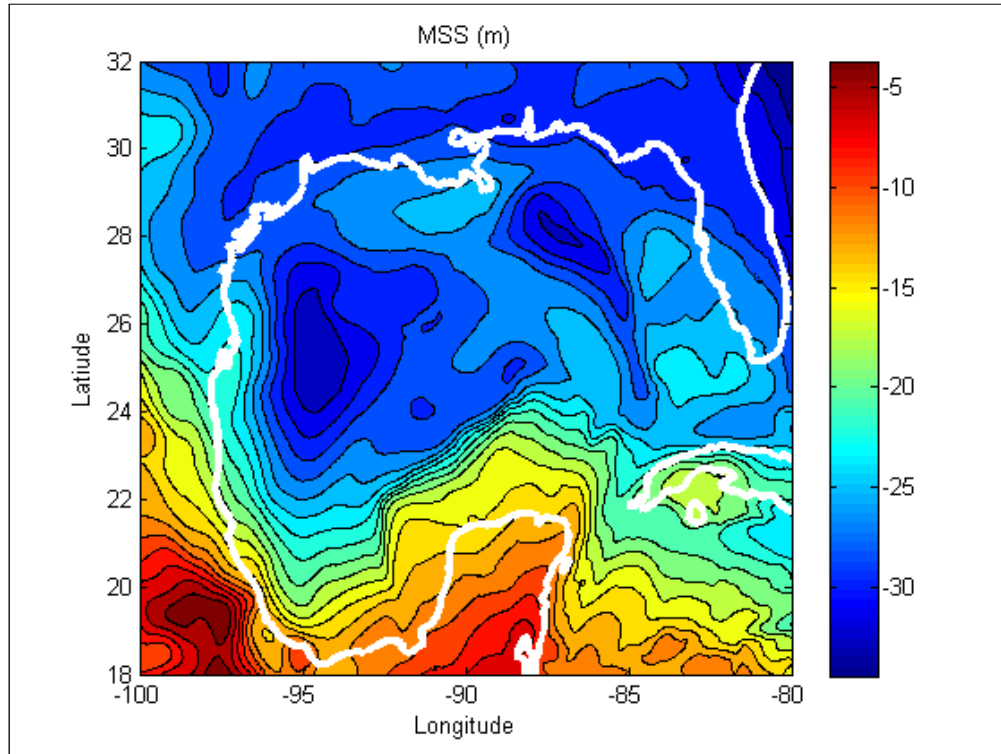
- The geoid is that horizontal equipotential surface that approximates Mean Sea Level (MSL), which, due to mean ocean dynamics, is not horizontal
- MSL is a tidal datum. It is the average water level observed at tide gauge over the 18.6 year precession of the lunar orbital plane w.r.t. the ecliptic plane
- A Mean Sea Surface (MSS) extends MSL over the entire sea surface by combining tide gauges with satellite altimetry
- Mean Dynamic Ocean Topography (DOT) is the difference between the geoid and MSS

6

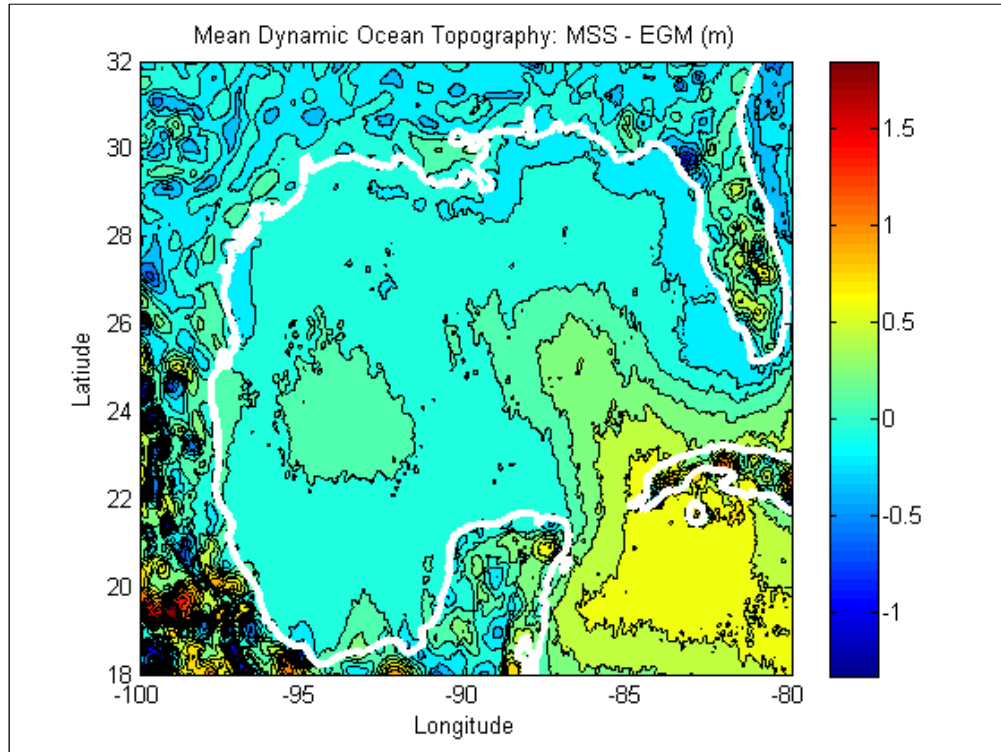
We'll see the lunar orbital plane in the extra slides.



Earth Gravity Model 2008 for the Gulf of Mexico



Mean Sea Surface for the Gulf of Mexico



Mean Dynamic Ocean Topography for the Gulf of Mexico

Tidal Constituents

- Tides are generated by the gravitational forces of the moon (68.5%) and the sun (31.5%)
- A constituent is a repeatable geometry in the positions of the earth, moon and sun expressed as a period (hours) and speed (degrees/hour)
- The tidal effect of a constituent at a specific place has an amplitude and phase shift w.r.t. some initial time
- The constituents with the greatest amplitude are:
 - M2, principal lunar semi-diurnal
 - S2, principal solar semi-diurnal
 - K1, diurnal lunisolar declination
 - O1, diurnal lunar declination

10

(Free comment on the notes above.)

A List of Tidal Datums

Approximate Lowest Astronomical Tide	Lower Low Water
Approximate Mean Low Water Springs	Lowest Low Water
Approximate Mean Low Water Tide	Low Water
Approximate Mean Low Water	Low Water Springs Low Water Springs
Approximate Mean Sea Level	Mean Higher High Water
Equinoctial Spring Low Water	Mean Higher Water
Half Tide	Mean High Water
Highest Astronomical Tide	Mean High Water Neaps
Higher High Water Large Tide	Mean High Water Springs
Highest Normal High Water	Mean Lower Low Water Springs
Higher High Water	Mean Lower Low Water
Highest High Water	Mean Low Water
High Water	Mean Low Water Neaps
High Water Springs	Mean Low Water Springs
Indian Spring High Water	Mean Sea Level
Indian Spring Low Water	Mean Tide Level
Lowest Astronomical Tide	Nearly Lowest Low Water
Lower Low Water Large Tide	Neap Tide
Lowest Low Water Springs	Spring Tide

11

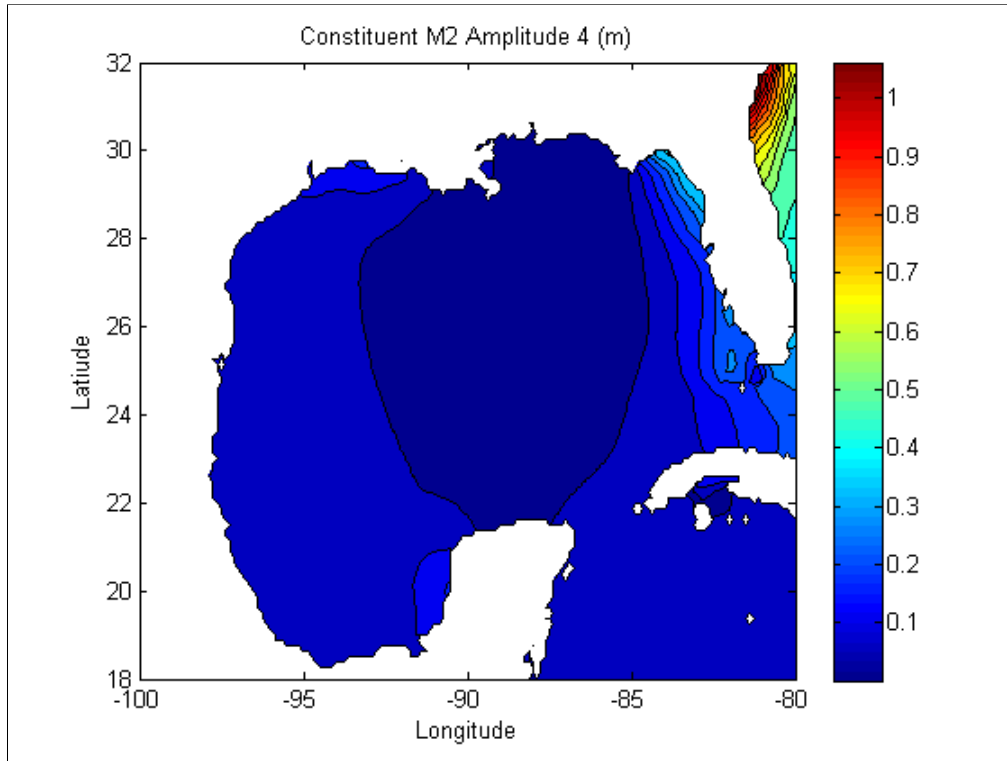
Here are a few tidal datums.

Mean Lower Low Water is actually an important tidal datum in the oil patch since offshore well logs in Canada and the US are referenced to MLLW.

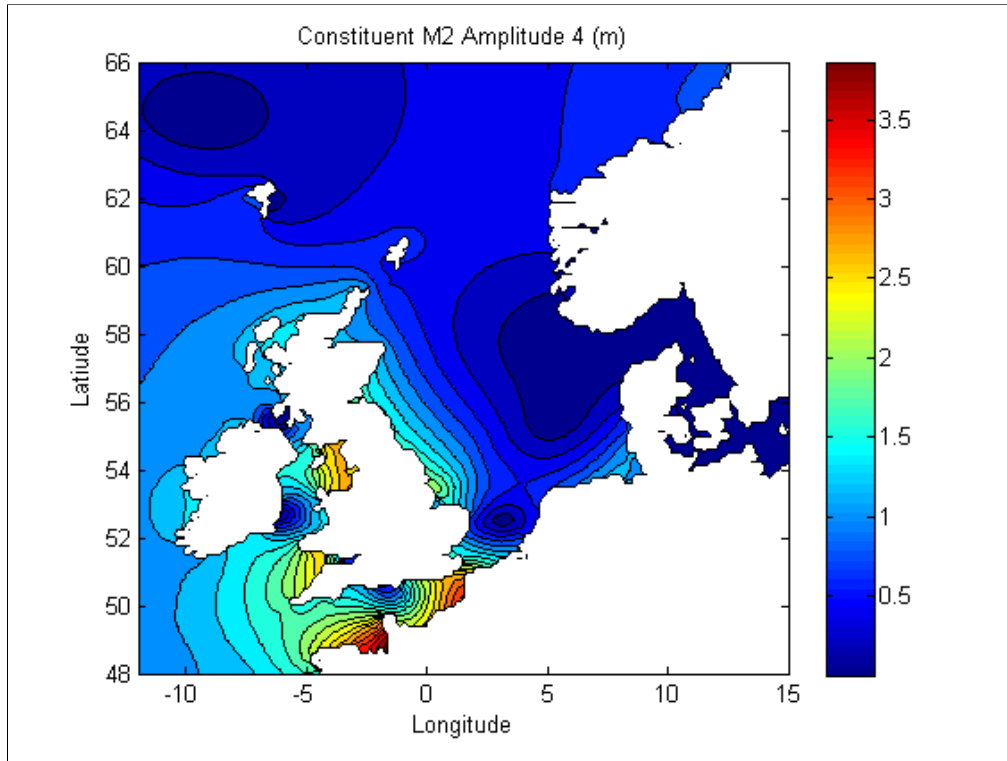
We'll talk more about Mean Sea Level.

Lowest Astronomical Tide (LAT) is important because it is chart datum in much of the world. LAT is the lowest tide celestially possible *ceteris paribus*. LAT may not be the lowest tide, however, since it doesn't consider weather, for example. LAT is used in seismic surveys, but it should be abandoned for that purpose in my opinion. Seismic depths aren't concerned with safe navigation. But it would be better if seismic depths at sea were related to seismic elevations on land. For that purpose, MSL is a better datum.

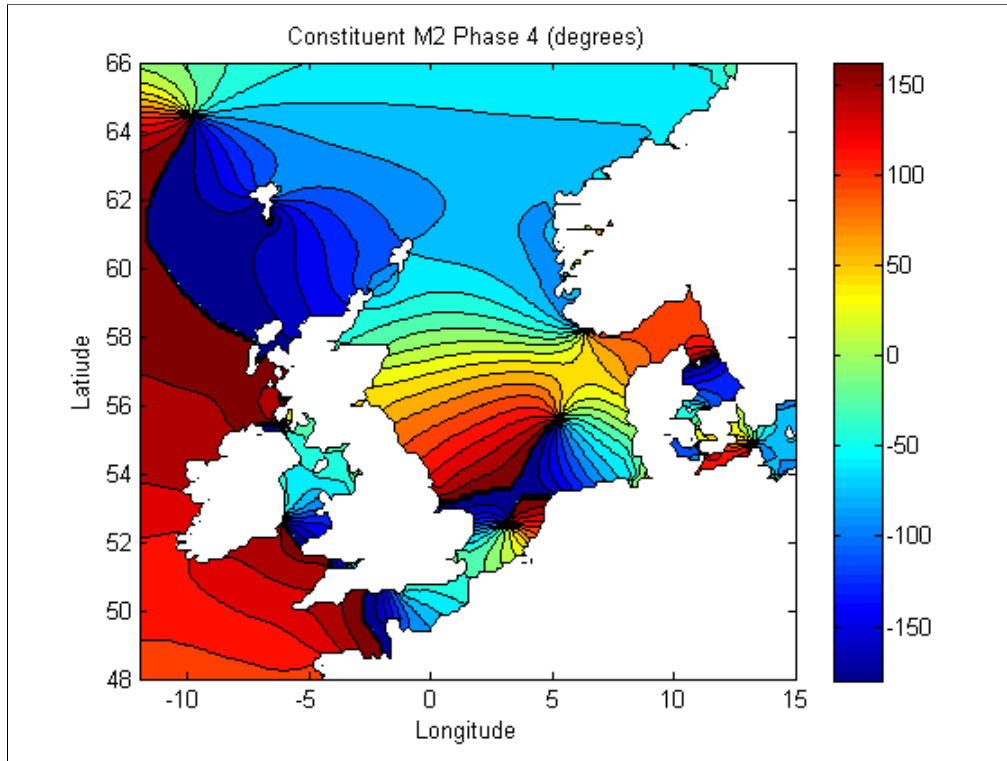
Like LAT, Indian Spring Low Water is another predicted (from constituents, i.e. not observed) tidal datum.



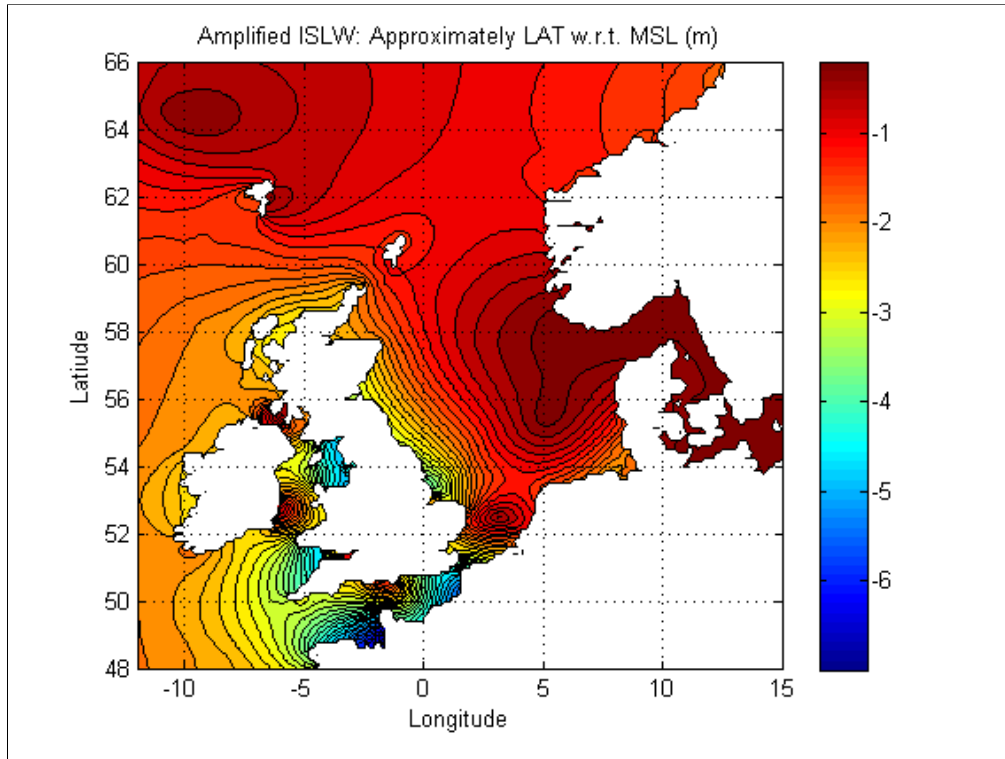
M2 is the constituent with the largest amplitude in the GOM



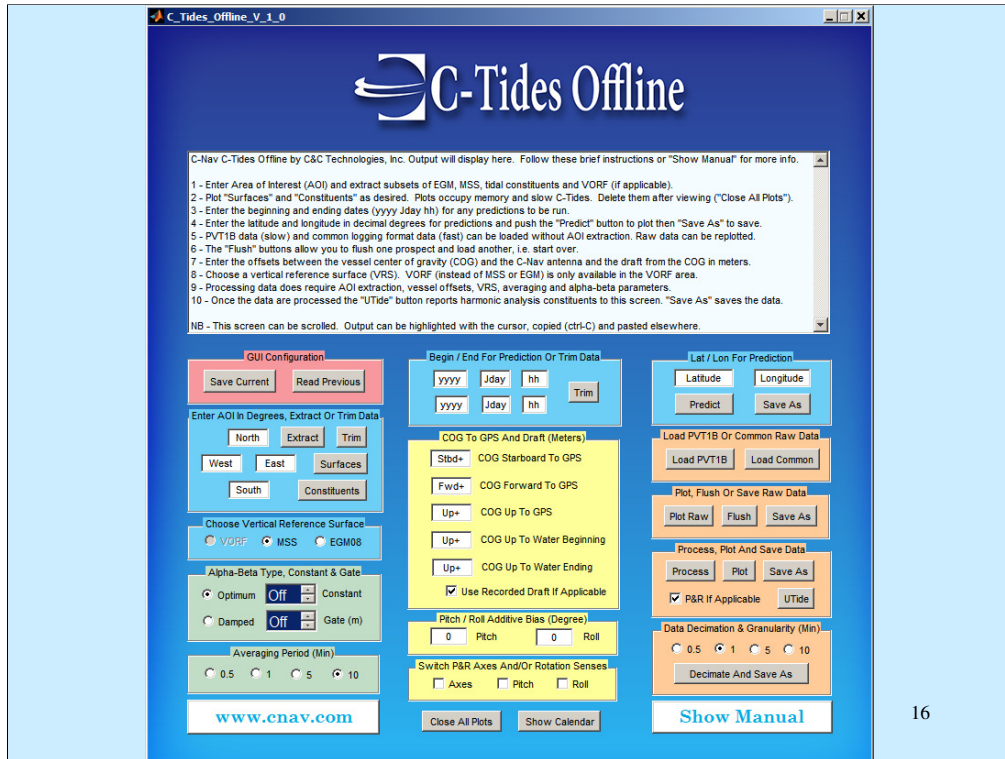
M2 amplitude in the North Sea



The phase of M2 in the North Sea

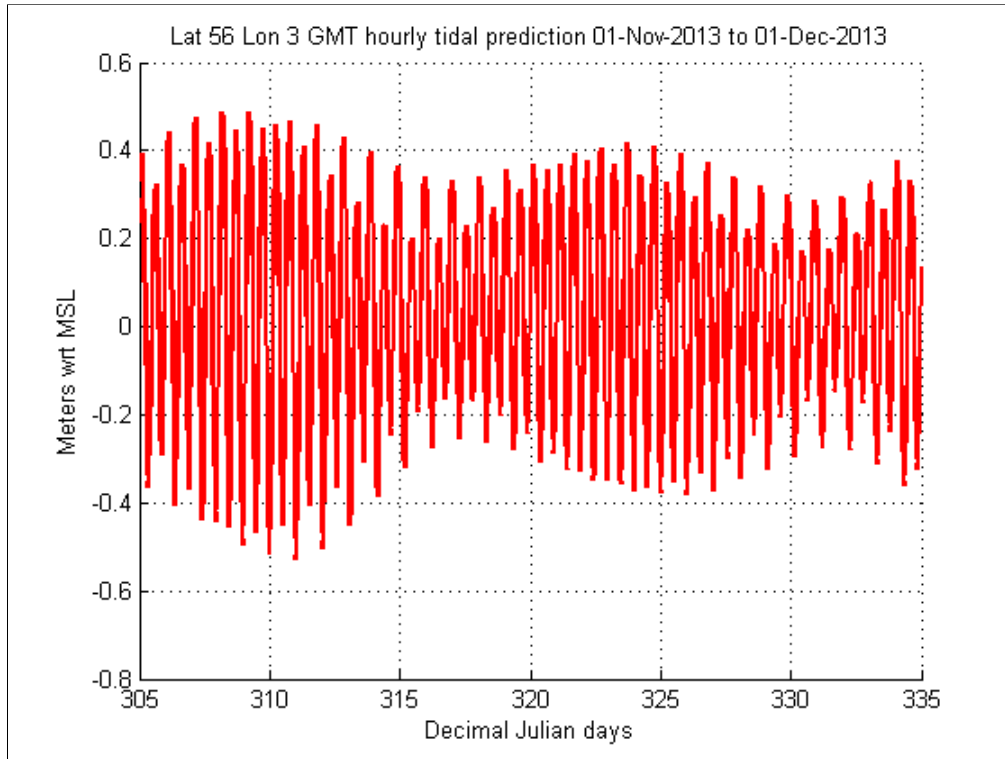


LAT w.r.t. MSL in the North Sea



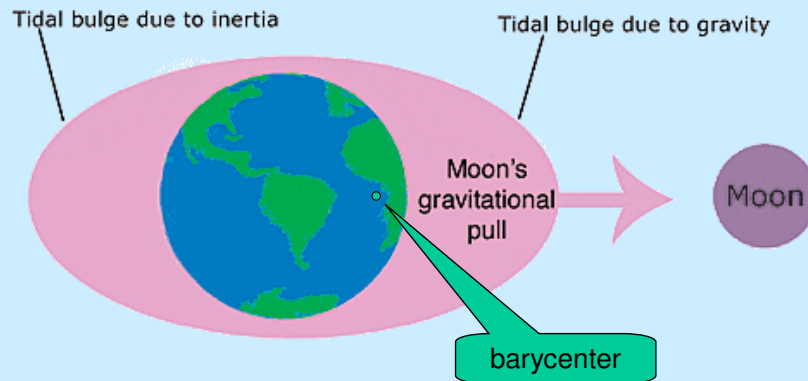
16

C-Tides. The source of the graphics. C-NAV by C&C Technologies



C-Tides predicted tides in the middle of the North Sea for the month of December 2013. Notice the diurnal tides. Notice that one high tide is higher than the other high tide that day. Notice the spring and neap tides. Today is day 324.

Tidal Hydrography



Because the earth-moon barycenter of rotation is not at the earth's center (actually $3/4$ earth radius from the center), water accelerates toward the moon at the sub-lunar point and away from the moon at its antipode, as shown, thus producing two high tides per day.

18

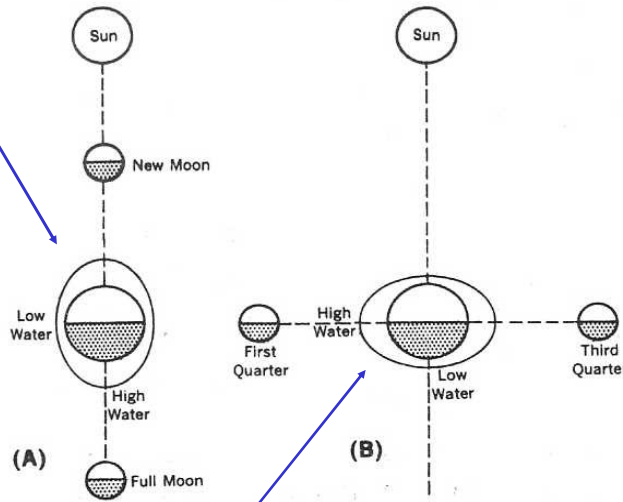
This graphic explains why most of the world has two high tides per day. The moon accounts for about two thirds of the tidal effect (the sun the other third). I've marked the barycenter on this graphic. It's the center of mass of the earth-moon system and it's about three quarters of an earth radius from the center of the earth. The moon rotates about the barycenter, not the earth center. There is also a motion of the earth around the barycenter, a wobble in the earth's orbit around the sun. The gravitational effect of the moon raises the tide on the side of the earth facing the moon. But because of earth motion about the barycenter, centrifugal force is greater on the side of the earth opposite the moon, like water in a bucket swung on a rope. This creates the second high tide.

Now, I have a question for you. If the moon were more massive such that the earth-moon barycenter moved into the space between the earth's surface and the moon, what would we call the moon?!?

That's right, we'd call it a sister planet, or a dual planet, or just a planet, not a moon. For a moon to be a moon (or a satellite), the planet-moon barycenter has to be within the major planet. Otherwise, it's a dual planet system.

Tidal Hydrography

Spring tides (higher high water / lower low water) occur when the earth and moon are aligned at new moon or full moon (A).



Neap tides (lower high water / higher low water) occur during the first and third quarters of the moon (B).

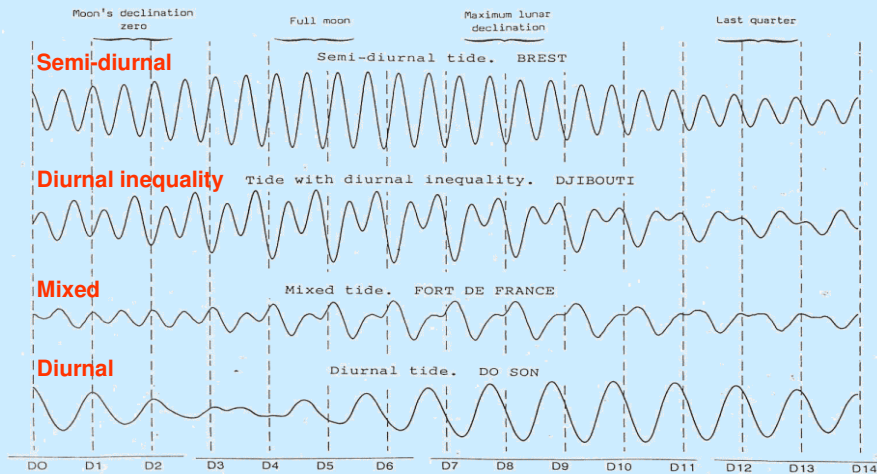
19

Spring tides (higher high water and lower low water) occur when the earth sun and moon are aligned. Neap tides (lower high water and higher low water) occur when the earth-moon and the earth-sun are perpendicular.

On my walks the last few mornings I've seen that the waning gibbous moon is approaching third quarter, so we should be nearing neap tide in a day or two.

(Thanks to Bowditch, American Practical Navigator, for this graphic.)

Tidal Hydrography



Example tides resulting from these celestial mechanics and **local geography** for different locations in the world over a 14-day period.

20

In addition to the positions of the moon and the sun, the tides are affected by the geographical distribution of land masses. This graphic from the French Manual of Hydrography exhibits that.

In Brest, France, which is on the open Atlantic south of Great Britain, we get two high tides per day, called semi-diurnal.


In Djibouti at the mouth of the Red Sea we also get two tides, but they are unequal.

At Fort de France on Martinique in the Caribbean the tide is mixed, some semi-diurnal and some diurnal.

At Do Son, a beach resort in the north of Vietnam on the South China Sea, the tide is diurnal, one high tide per day.

The interplay of celestial mechanics and the earth's geography is complex.

(This graphic is from the French Manual of Hydrography.)

Bedford Institute of Oceanography  Canada

BIO Science

BIO Home > Research > Ocean Science > WebTide Tidal Prediction Model (v0.7.1)

WebTide Tidal Prediction Model (v0.7.1)

Usage Instructions
WebTide News!

Data Sets

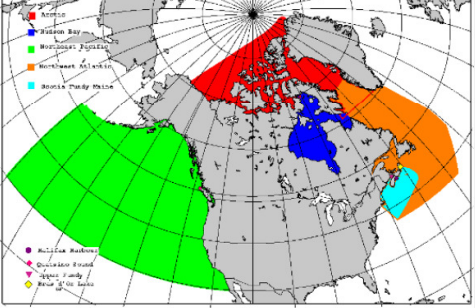
- Arctic
- Halifax Harbour
- Hudson Bay
- Northeast Pacific
- Northwest Atlantic
- Quatsino Sound
- Scotia-Fundy-Maine
- Bras d'Or
- Upper Fundy
- Global

Log Files

- Change Log
- Windows Trouble Shooting
- Linux/Unix Trouble Shooting

WebTide is a graphical user interface for a tidal prediction program. Predictions can be obtained for any point inside a selected model domain and are based on analyses of ocean model output.

Potential users should be aware that this is still a research product and exercise due caution. This product should not be used for navigation purposes.



Download Data Sets:

- Arctic
- Bras d'Or Lake
- Halifax Harbour
- Hudson Bay
- Northeast Pacific
- Northwest Atlantic
- Quatsino Sound
- Scotia - Fundy - Maine
- Upper Fundy
- Global Data

Download Log Files:

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	includes Java VM	without Java VM	Instructions
Windows	Download (26.2M)	Download (10.6M)	View
Linux	Download (30.8M)	Download (10.5M)	View

WebTide is a free alternative to C-Tides. It does not use as many constituents, however.

MSL to Geoid to Vertical Datum

- The geoid (W_0) approximates as Mean Sea Level
- On land the geoid is the level that MSL would take “if tunnels were bored through the land”
- The geoid is realized as a vertical datum on land
- A vertical datum is defined by the elevations of benchmarks (monuments) embedded in the land

22

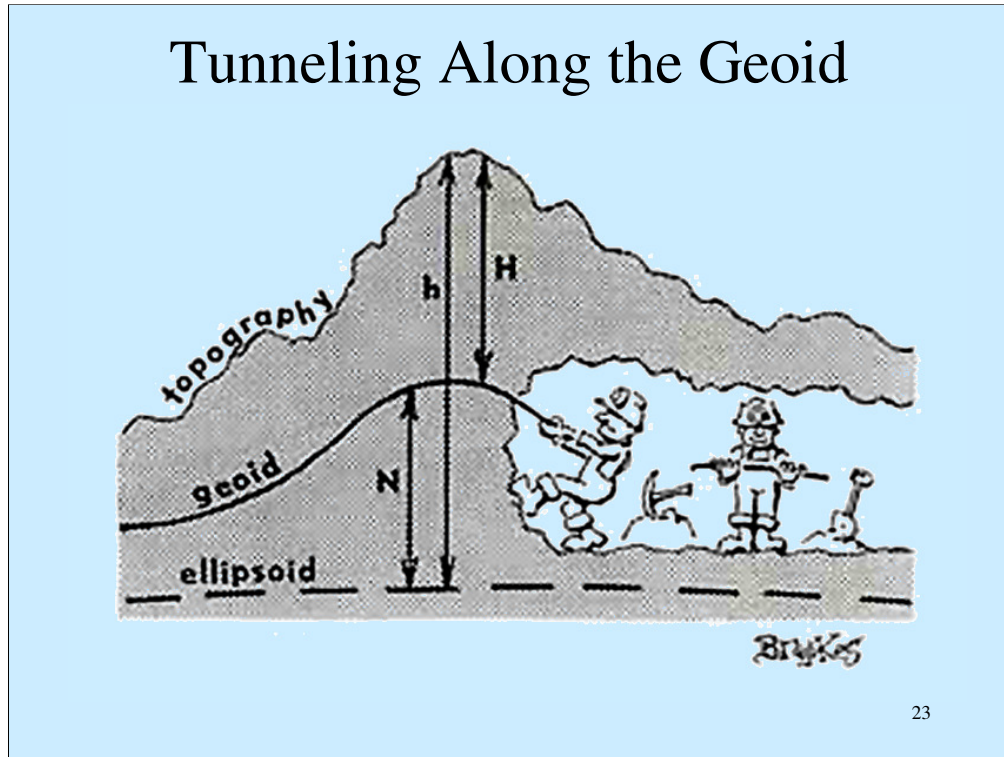
The geoid is the equipotential surface that approximates Mean Sea Level over the seas.

The geoid on land is the level that MSL would take if it could.

The geoid is at the foundation of vertical datums on land, which are realized in the form of monuments called benchmarks.

How many times have we heard the expression “if tunnels were bored through the land.

Tunneling Along the Geoid



Well, here are two geodesists boring a tunnel through the land and following the geoid! You'll notice that their feet are a bit wet as MSL flows in.

This graphic nicely depicts the three surfaces of interest, the topographic surface on which we walk, the geoid and the ellipsoid. The topography and the geoid are physical surfaces that can be “observed” with simple instruments, our feet in the case of the topography and a plumb bob or a carpenter’s level in the case of the geoid. The ellipsoid is a mathematical abstraction determined by the adjustment of many surveying and astronomical measurements. It cannot be observed simply (unless, of course, we regard a GNSS receiver to be a “simple” instrument).

We also have three vertical measurements on this graphic, the ellipsoidal normal (h), the gravitational vertical (H) and the geoidal separation (N). Because these three are not necessarily co-linear, it’s not necessarily true that $h = N + H$ exactly, but very close.

(Thanks to the NGS for this graphic.)

Vertical Datum Benchmark

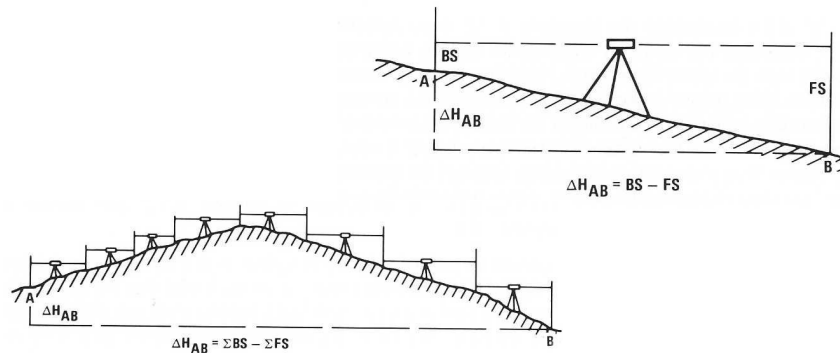


A benchmark is a brass cap embedded in concrete, the physical embodiment of a vertical datum. **A benchmark will have different elevations depending upon the vertical datum referenced.**

24

(Thanks to the NGS for this graphic.)

Spirit Leveling



Spirit leveling is surveying in the vertical with an instrument (called a spirit level) that transfers elevations from tidal stations on one coast over the land to a tidal station on another coast in a “level net” of “level lines” with benchmarks in the ground. **At key benchmarks gravity measurements are also made.**

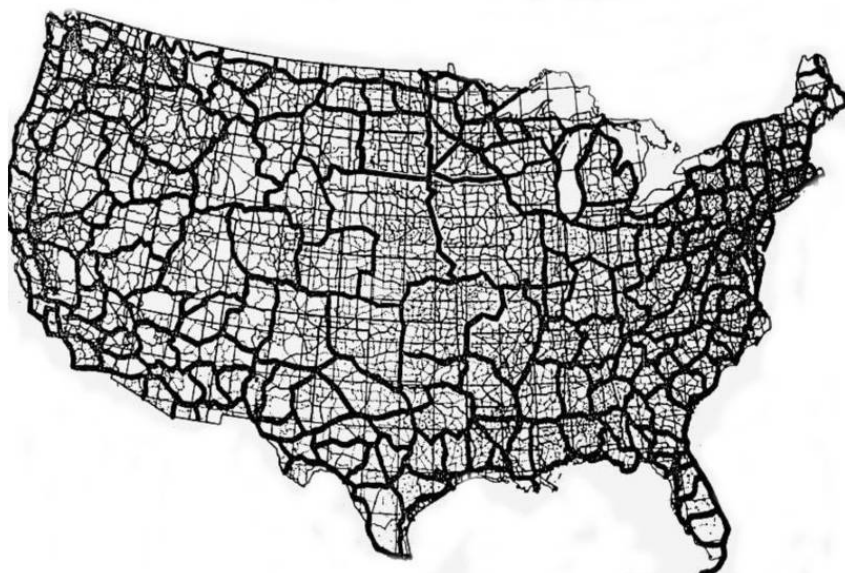
25

Early in my career I did trigonometric leveling with a theodolite by measuring vertical angles and accounting for curvature and refraction. But I never did spirit leveling with a level that observes only in the horizontal. Spirit leveling is slow going on an incline as shown here, since the fore shot and the back shot need to be balanced to cancel the effects of curvature and refraction. Your progress on an incline is limited by rod height.

Gravity measurements are not necessary for local leveling, such as at a construction site, but they are critical for the establishment of a vertical datum.

(Thanks to Gregory Hoar for this graphic.)

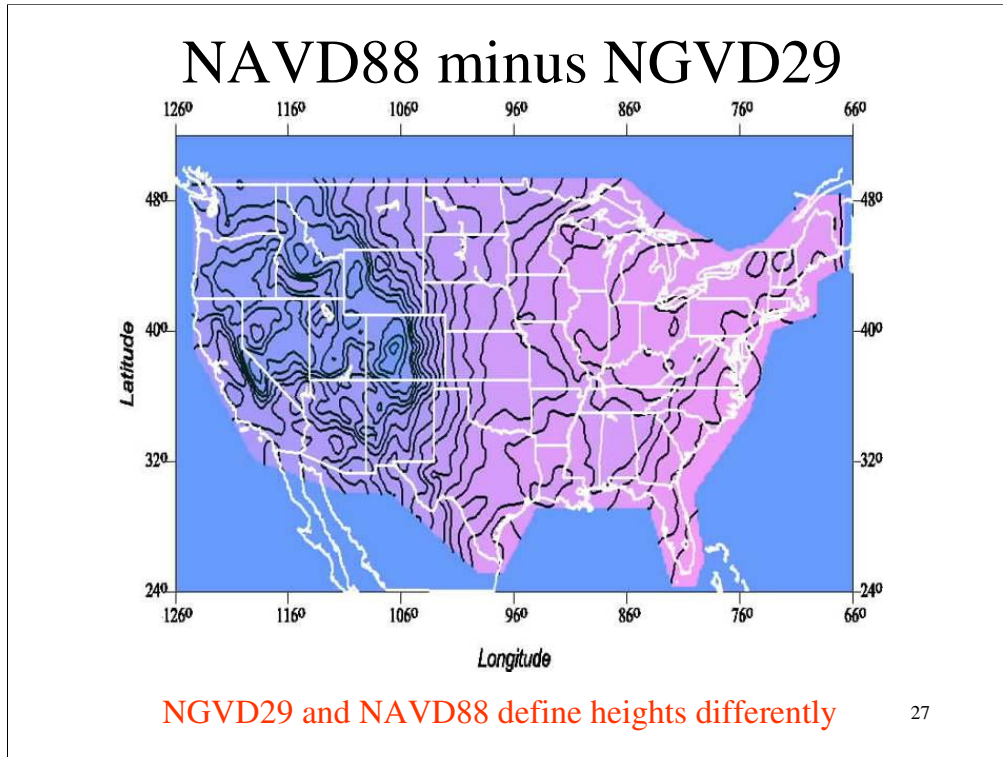
NAVD88 Level Lines



26

This graphic depicts the level lines that comprise the North American Vertical Datum 1988 (NAVD88) in the US. (The datum extends into Canada and Mexico.) There's a lot. Gravity stations are not shown.

(Thanks to NGS for this graphic.)



This contour plot shows the differences between NAVD88 and its predecessor, NGVD29 (National Geodetic Vertical Datum 1929), in the US. Since the contours are not labeled, I can tell what I want! What I will tell you is that NGVD29 is a bit higher in the East up to a maximum difference of about half a meter in Florida. NAVD88 is higher west of the Rockies up to about 1.5 meters. So there is a total difference of about 2 meters from east to west.

We will see later that the definition of orthometric height is different between NGVD29 and NAVD88.

(Thanks to NGS for this graphic.)

Some Vertical Datums Elsewhere

- Australian Height Datum
- Bandar Abbas, Fao (Iran)
- Caspian (Azerbaijan, etc.)
- DHHN85 & 92 (Germany)
- EGM2008 (Whole World)
- EVRF2000 (Europe)
- IGLD 1985 (Great Lakes)
- KOC WD (Kuwait Wells)
- Kuwait PWD (Kuwait)
- Lagos 1955 (Nigeria)
- Lerwick (UK)
- MSL (Oceans of the World)
- NGVD29, NAVD88 (USA)
- NN54 (Norway)
- Normaal Amsterdams Peil (Netherlands)
- PDO Height Datum 1983 (Oman)
- Yellow Sea, Yellow Sea 1956 & 1985 (China)
- The EPSG dataset lists more than 128 vertical datums worldwide. Some countries have more than one vertical datum.

28

Here are some vertical datums elsewhere. The EPSG dataset of the OGP currently lists 128 vertical datums worldwide.

Most vertical datums are within a couple meters of the geoid/MSL. The Caspian is an exception. It's about 28 meters lower than MSL. Also, the Kuwait datum differs by about 5 meters.

Later we'll talk a bit more about the International Great Lakes Datum (IGLD) of 1985.

Some of you will be familiar with Lerwick in the UK.

Normaal Amsterdams Peil is used elsewhere in Europe than The Netherlands.

Vertical Datum Modeling

- The US National Geodetic Survey provides software applications to transform among NGVD29, NAVD88, the geoid, tidal datums and NAD83 ellipsoid height
 - VERTCON, GEOID09, GEOID03, USGG2003, GEOID99, G99SSS, G99BM, CARIB97, MEXIC097, DMEX97, VDatum and **GRAV-D (2022)**
- Other governments provide similar software for their vertical datums
- In the absence of country-specific modeling software, **Earth Gravity Model 2008 (EGM08)** can be applied to orthometric heights to produce WGS84 ellipsoidal heights ... or vice versa

29

Here's a list of some of the vertical datum modeling software that is North America specific. Governments in other parts of the world supply similar software.

In the absence of country-specific modeling software, Earth Gravity Model 2008 (EGM08) can be applied to orthometric heights to produce WGS84 ellipsoidal heights ... or vice versa. In fact, if world-wide consistency is desired, then EGM08 may be preferable to country-specific models.

One important reason to turn orthometric heights into ellipsoidal heights is to turn ellipsoidal coordinates into Earth-Centered Earth-Fixed (ECEF) coordinates for 3D visualization.

Vertical Datum Types

- The vertical is defined differently in different vertical datum types
- Common definitions are:
 1. Geopotential number C_p in GPUs
 2. True orthometric heights (a variation of C_p)
 3. Dynamic heights (a variation of C_p)
 4. Normal orthometric heights (a variation of C_p)
 5. Helmert orthometric heights (a variation of C_p)
 6. Normal heights

30

The vertical is defined differently in different vertical datum types. Central to most of these definitions is the geopotential number (C_p) in the geopotential units (GPU) that we discussed earlier.

One vertical datum type is just the use of geopotential numbers for height, that is, not a linear unit (like meters).

True orthometric heights, dynamic heights, normal orthometric heights and Helmert orthometric heights are all variations of the geopotential number and some form of gravity.

And then there are normal heights, which don't seem very normal to me.

(1) Geopotential Numbers

- Geopotential numbers (C_P) are expressed in geopotential units (GPUs) seen earlier
- If W_0 is the geopotential of the geoid, W_A the geopotential of point A, h the height of point A and g is the acceleration of gravity, the geopotential number of point A is:

$$C_P = W_0 - W_A = \int_0^A g(\phi, h) \cdot dh$$

31

The easiest way to get the geopotential number of point A is to use the formula on an earlier slide to determine the geopotential in GPUs of point A and difference that with the geopotential of the geoid. The term farther to the right is the mathematical inverse of the definition of g (acceleration of gravity) as the gradient of W (geopotential).

(1) Geopotential Numbers

- The geopotential unit (GPU) of C_p is:
 - 1 GPU = 10 meter²/second² = 1 kgal·meter
- Advantages:
 - All points on water have the same GPU
 - Spirit leveling loops will “close” regardless of route
- Disadvantages:
 - GPU differs numerically from elevations by 2%
 - GPU is an unusual unit (kgal-meter)

32

We’ve already defined a GPU as 10 meters squared per second square. That works out to be a kilogal-meter. A gal (for Galileo) is the CGS unit (not SI, or MKS) for gravity. It’s an acceleration with units of centimeter per seconds squared. A gravity meter typically provides readings in milligals, a thousandth of a gal.

One of the advantages of the geopotential number is that it follows the equipotential surface. Therefore, all points on the surface of a large lake at elevation above the geoid will have the same geopotential number. It is a shocking reality that in the northern hemisphere the orthometric height of the northern shore of such a lake will be lower than the orthometric height of the southern shore.

Disadvantages are that the GPU is a strange unit (meters squared per seconds squared) and numerically it differs from orthometric height by about 2%. For example, 100 GPUs is about 102 meters.

(2) True Orthometric Heights

- **But ...** true orthometric heights result by dividing the C_p by mean gravity along the plumb line from the topography to the geoid
- **But ...** since mean gravity cannot be measured, in most vertical datums the C_p is divided by **some estimate or function of surface gravity** in order to approximate an orthometric height

33

A true orthometric height is defined by dividing the geopotential number in GPUs by mean gravity along the plumb line from the topography to the geoid. But unless we have a convenient (and dry) borehole and a gravity meter, we don't know mean gravity. So we estimate or use some function of surface gravity. Different estimates and functions lead to different kinds of orthometric height.

(3) Dynamic Heights

- Dynamic heights result by dividing geopotential numbers by the same standard gravity value
- For example, the **International Great Lakes Datum of 1985** (IGLD 1985, US and Canada) uses dynamic heights and an acceleration of gravity of 9.806199 m/s^2 defined at 45° North on the GRS80 ellipsoid (NAD83)
- Dynamic heights preserve water levels, but they differ slightly from orthometric heights

34

(No notes.)

(4) Helmert Orthometric Heights

- Helmert orthometric heights result by dividing geopotential numbers by an estimate of the mean gravity along the plumb line
- The estimate is called **Prey reduction** and it is based upon surface gravity (g) and an assumed crustal density and free-air gravity gradient

$$H = C_p / (g + 0.0424H)$$

- **NAVD88** is in Helmert orthometric heights

35

Notice that the equation for height is recursive, H is on both the left and the right. NAVD88 is in Helmert orthometric heights.

Many of you tonight will Google Prey reduction and you're in for a surprise. It turns out that Prey is a rock band with a recent album called Reduction. You'll learn more about contemporary music than geodesy unless you search deeply!

(5) Normal Orthometric Heights

- Normal orthometric heights result by dividing geopotential numbers by “normal gravity”
- Bomford gives this formula (with modified units) for normal gravity as a function of latitude, height and the radius of the earth (R):

$$\gamma_h = 9.78(1 + 0.0053 \sin^2 \phi - 2h / R) m / s^2$$

- **NGVD29** is in normal orthometric heights

36

The formula for normal gravity given here is similar to the formula for the orthometric correction (based upon normal gravity) that one needs to use when spirit leveling. The orthometric correction is covered in the extra slides at the end. NGVD29 is in normal orthometric heights.

(6) Normal Heights

- Normal heights are based upon the gravity theory of Molodensky and upon two additional earth surfaces, the quasi-geoid and the telluroid
- Normal heights are used in the former Soviet Union countries, in Eastern Europe, and are being adopted in Western Europe
- Normal heights correspond to orthometric heights over the oceans and at low elevations, but differ by up to 2 meters in high elevations

37

If you thought the topographic, geoidal and ellipsoidal surfaces were enough, normal heights based upon the gravity theory of Molodensky introduce two more surfaces, the quasi-geoid and the telluroid.

And if you think the telluroid is a ski resort in Colorado, you're wrong! Fortunately, Vanicek and Krakiwsky have decent explanations of the quasi-geoid and the telluroid and I'll defer to them.

Why Should I Care?!?

- Sun/Moon \Rightarrow Tides \Leftrightarrow Geoid \Leftrightarrow Vertical Datums
- Vertical datums types are well described
- Including vertical datum audits along with horizontal datum audits is desirable in the oil patch
- Modeling software transforms between orthometric and ellipsoidal heights
- Ellipsoidal height, latitude and longitude are just a conversion from ECEF (Earth-Centered Earth-Fixed)
- ECEF supports undistorted visualization in 3D

38

We've discussed the sun and the moon and seen how they cause the tides. The geoid is the equipotential surface defined by a specific tidal datum. The geoid in turn is at the foundation of most types of vertical datums.

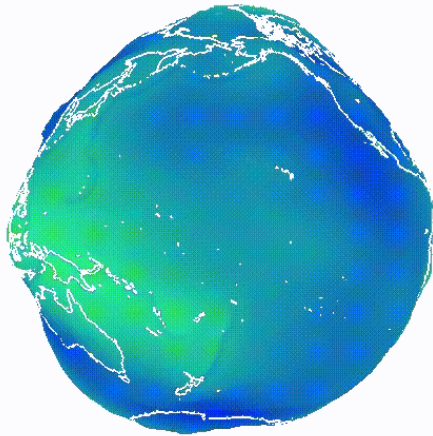
These vertical datums are well described and their descriptions need to find their way into our audit trails.

Modeling software allows us to transform between orthometric and ellipsoidal heights.

Ellipsoidal coordinates are just a transformation away from ECEF.

Once in ECEF we can visualize the world in 3D with geodetic rigor and without cartographic distortion.

Here's Why: Undistorted 3D Visualization



39

This animated TIFF depicts EGM08 in ECEF exaggerated 10,000 times for visual effect. The vertical is distorted to make the point that the vertical fits as well into ECEF as the horizontal. In ECEF we can rotate our 3D image like a globe in our hands in order to eliminate the cartographic distortion that all 2D maps are heir to.

References

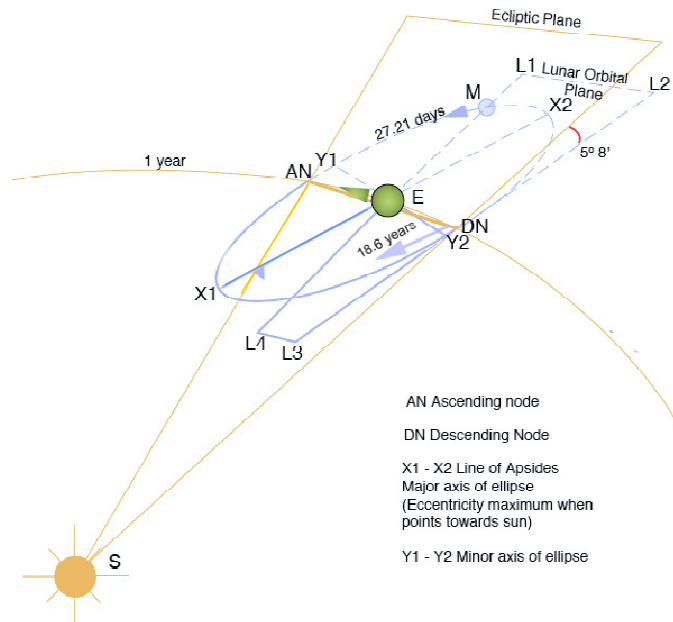
- Bomford, “Geodesy”, 1980
- Bowditch, “American Practical Navigator”, 1977
- Hoar, “Satellite Positioning”, Magnavox, 1982
- Service Hydrographique et Oceanographique de la Marine, “Manual of Hydrography”, 1984
- Vanicek and Krakiwsky, Geodesy: The Concepts, 1987
- Wikipedia
- Graphics from various DMA/NIMA/NGA and NOAA/NGS presentations on the web
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40

(No notes.)

Extra Slides

Lunar Nodal Period of 18.6 Years



The Lunar Orbital Plane (LOP) is inclined with respect to the Ecliptic Plane (EP) by $5^{\circ}8'$. The LOP rotates inertially with respect to the EP in an 18.6 year period.

Therefore, to catch all possible positions of the moon, one must observe the tides for 18.6 years.

42

I mentioned that the tidal observation period is 18.6 years. Here's why.

We have here the sun, the earth and the moon. The orbit of the earth around the sun is called the ecliptic. It's shown here as the yellow triangle. The orbit of the moon about the earth is not in the same plane as the ecliptic. The moon's orbital plane is inclined a little more than 5 degrees from the ecliptic, which accounts for the different elevations of the moon. This is also why lunar and solar eclipses are relatively rare. Here we see the ascending (AN) and descending (DN) nodes of the lunar orbit in line with the earth's orbit. But it could be otherwise. In fact, those nodes rotate almost 20 degrees per year with respect to the ecliptic, or 18.6 years for a full rotation.

Therefore, it takes 18.6 years for the tides to experience all possible positions of the moon.

(This graphic is from Wikipedia.)

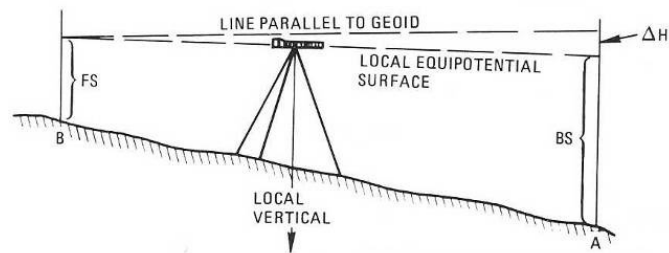
Normal Orthometric Heights: An Alternative Approach

- In the absence of geopotential numbers, normal orthometric heights can be computed by applying the “orthometric correction” to a spirit level line from a known benchmark
- The orthometric correction is also based upon normal gravity and is exhibited in the next slide

43

(No notes.)

Orthometric Correction



Hoar gives the following for the orthometric correction (ΔH), where H is the average elevation of the line, ϕ is the average latitude of the line and $\Delta\phi$ is the difference in latitude in radians

$$\Delta H = H(0.0053 \sin 2\phi)\Delta\phi$$

44

(Thanks to Gregory Hoar for this graphic.)

Vertical Datums in North America

National Geodetic Vertical Datum of 1929 [NGVD 1929]

A fixed reference adopted as a standard geodetic datum for elevations determined by leveling. The datum was derived for surveys from a general adjustment of the first-order leveling nets of both the United States and Canada. In the adjustment, mean sea level was held fixed as observed at 21 tide stations in the United States and 5 in Canada. The year indicates the time of the general adjustment. A synonym for Sea-level Datum of 1929. The geodetic datum is fixed and does not take into account the changing stands of sea level. Because there are many variables affecting sea level, and because the geodetic datum represents a best fit over a broad area, the relationship between the geodetic datum and local mean sea level is not consistent from one location to another in either time or space. For this reason, the National Geodetic Vertical Datum should not be confused with mean sea level.

North American Vertical Datum of 1988 [NAVD 1988]

A fixed reference for elevations determined by geodetic leveling. The datum was derived from a general adjustment of the first-order terrestrial leveling nets of the United States, Canada, and Mexico. In the adjustment, only the heights of the primary tidal bench mark, referenced to the International Great Lakes Datum of 1985 (IGLD 1985) local mean seal level height value, at Father Point, Rimouski, Quebec, Canada was held fixed, thus providing minimum constraint. NAVD 1988 and IGLD 1985 are identical. However, NAVD 1988 bench mark values are given in Helmert orthometric height units while IGLD 1985 values are in dynamic heights.

45

(No notes.)